QMath 2016

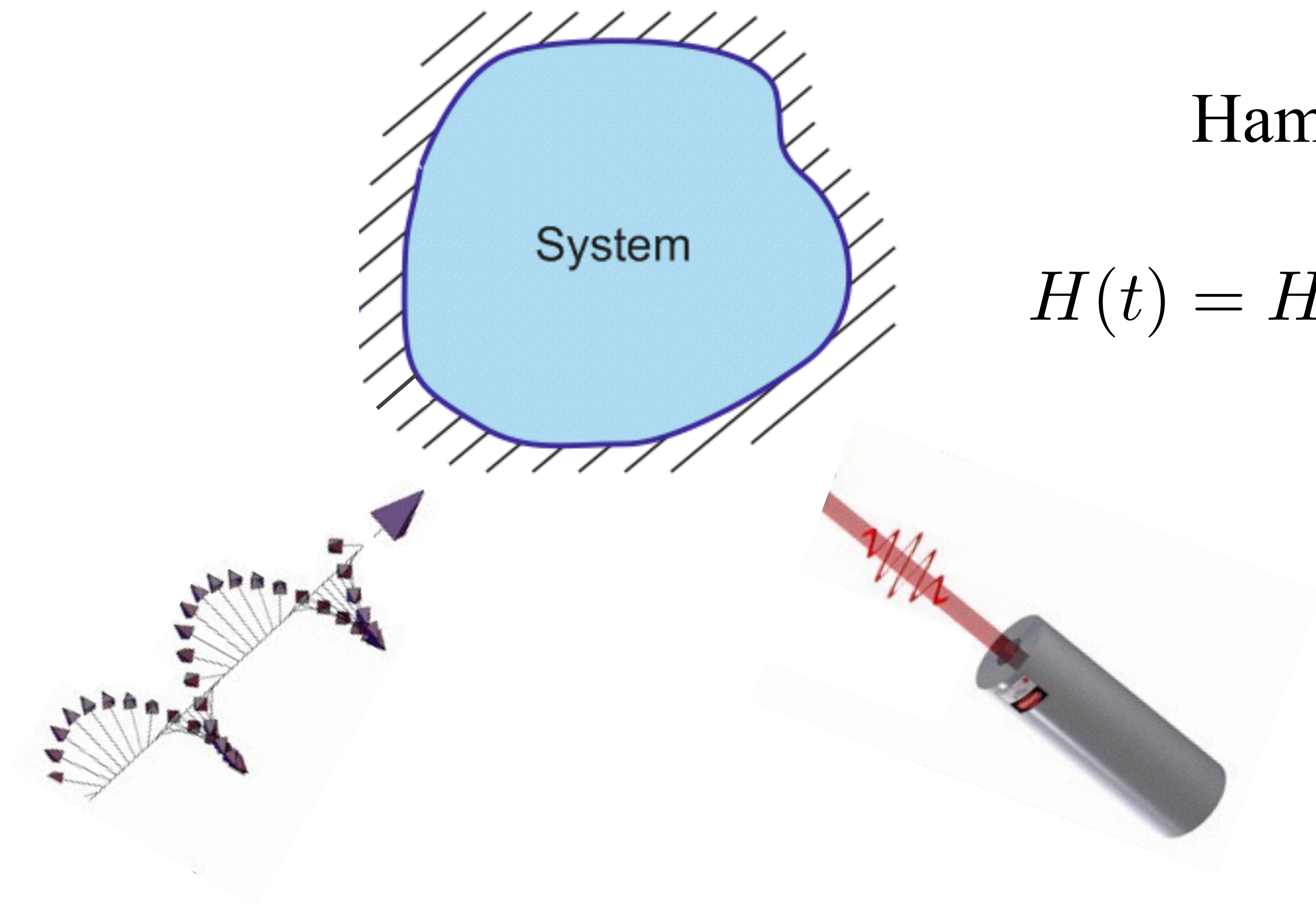
Heating in periodically driven Floquet systems

Anushya Chandran

Boston University

Floquet system

Periodically driven isolated system

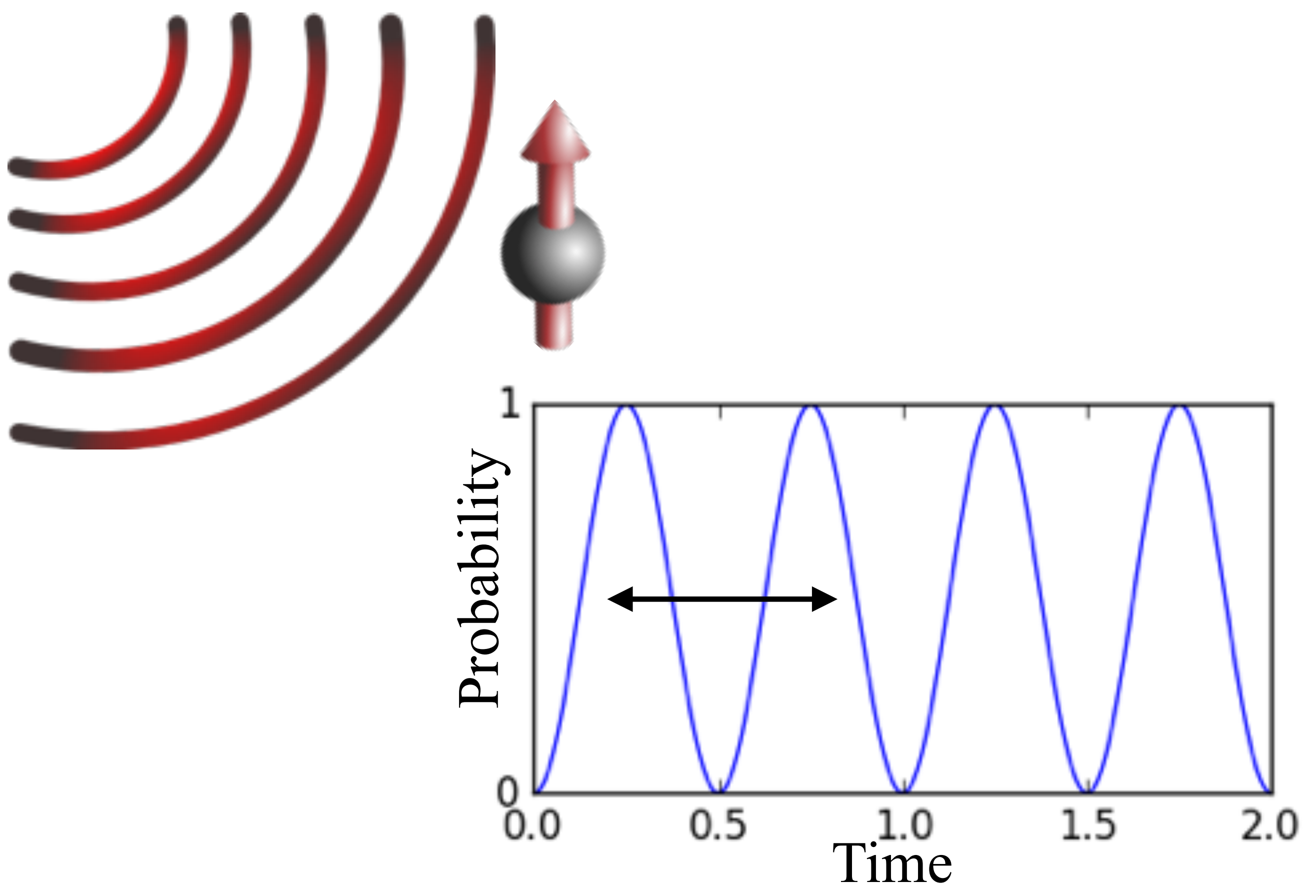


Hamiltonian H_0

$$H(t) = H_0 + V\cos(\omega t) H_1$$

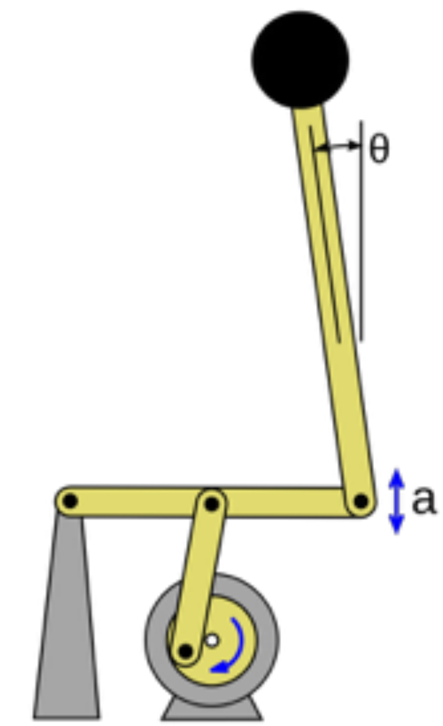
Few-body Floquet systems

Rabi oscillations



Amplitude of drive ➡ Frequency

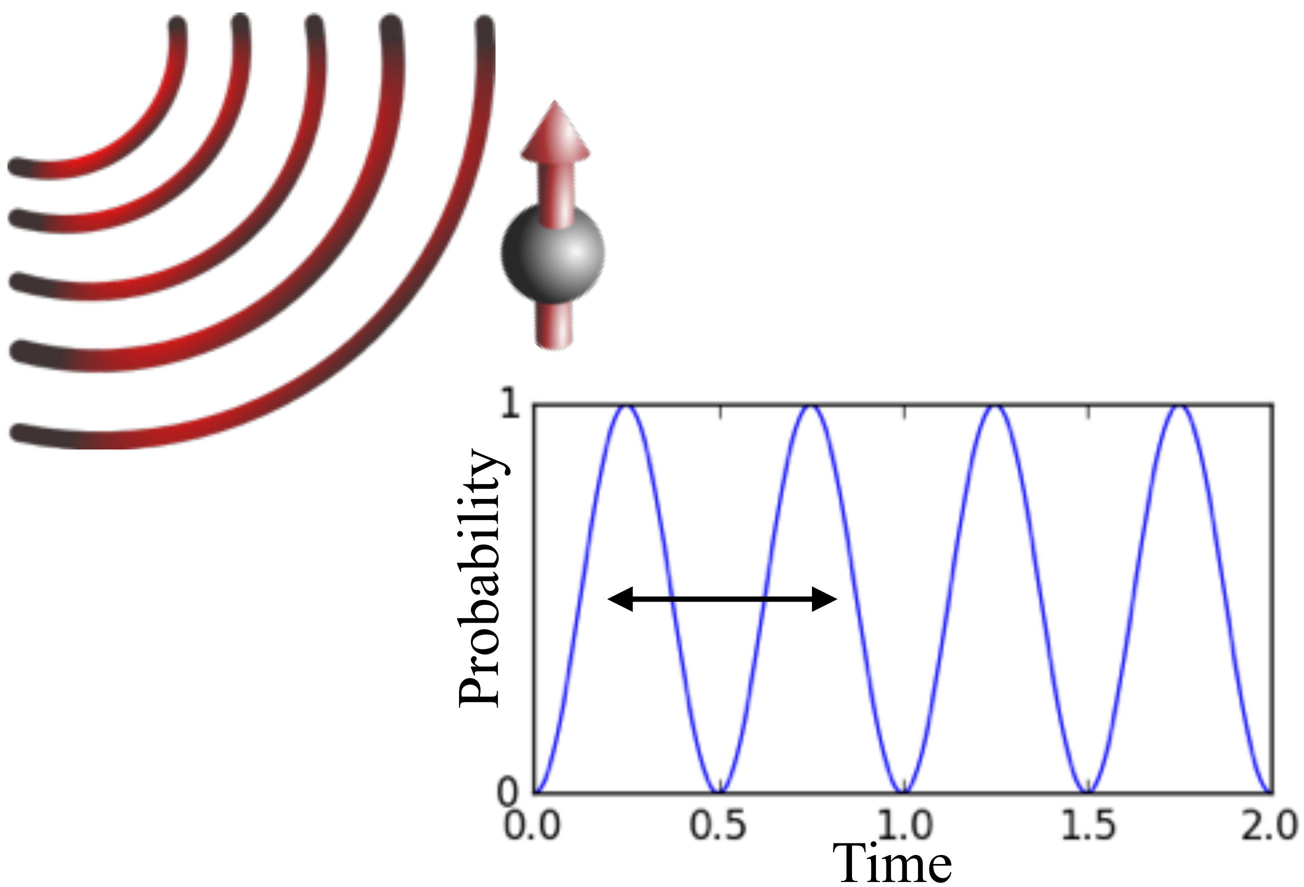
Kapitza pendulum



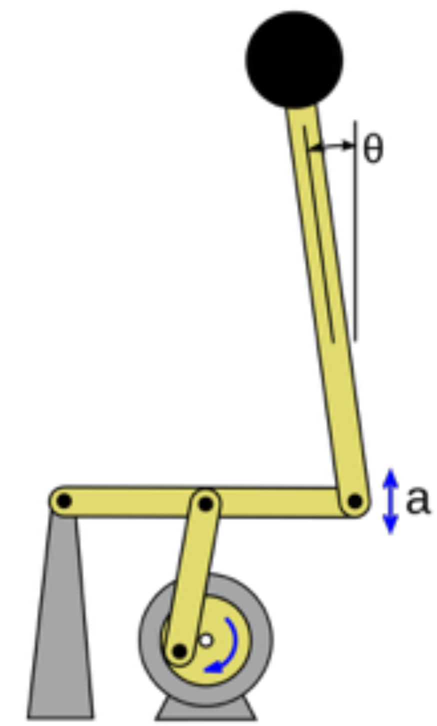
New stable equilibrium

Few-body Floquet systems

Rabi oscillations



Kapitza pendulum

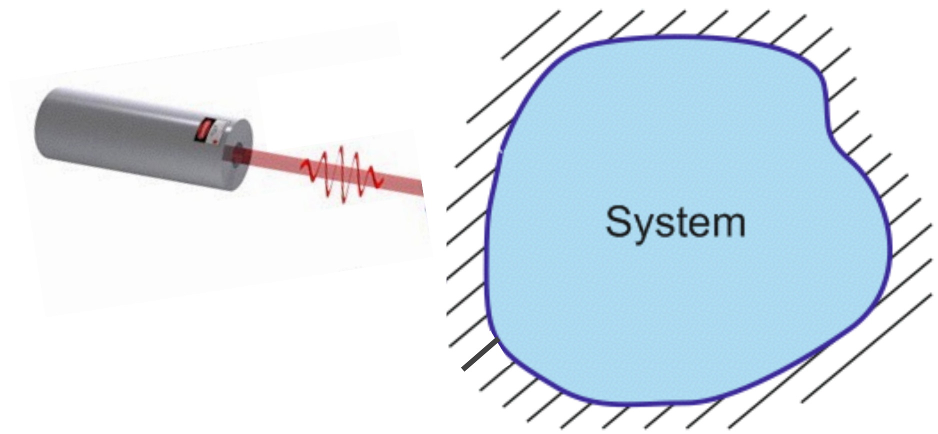


Amplitude of drive ➡ Frequency

New stable equilibrium

Many-body?

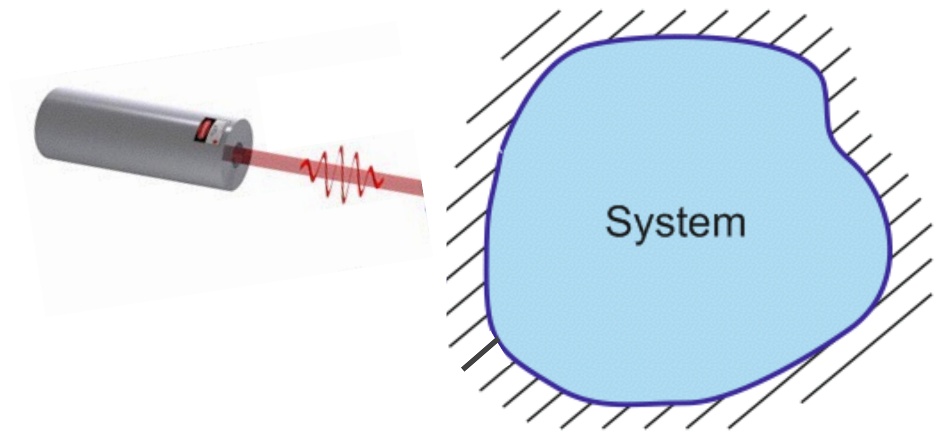
Interest: fundamental & engineering



Simplest non-equilibrium setting: what can happen?

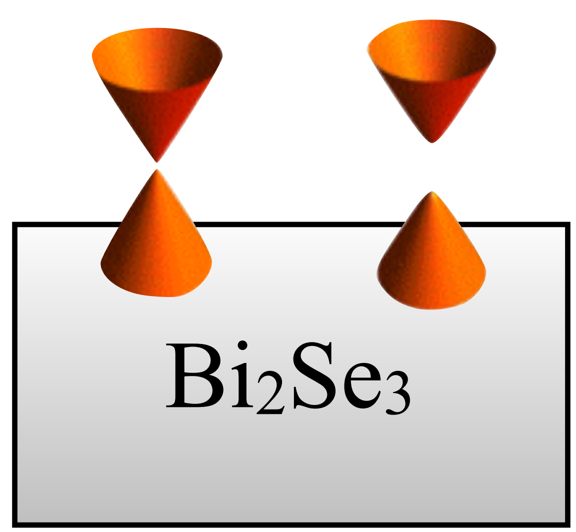
Engineer new states out of equilibrium?

Interest: fundamental & engineering

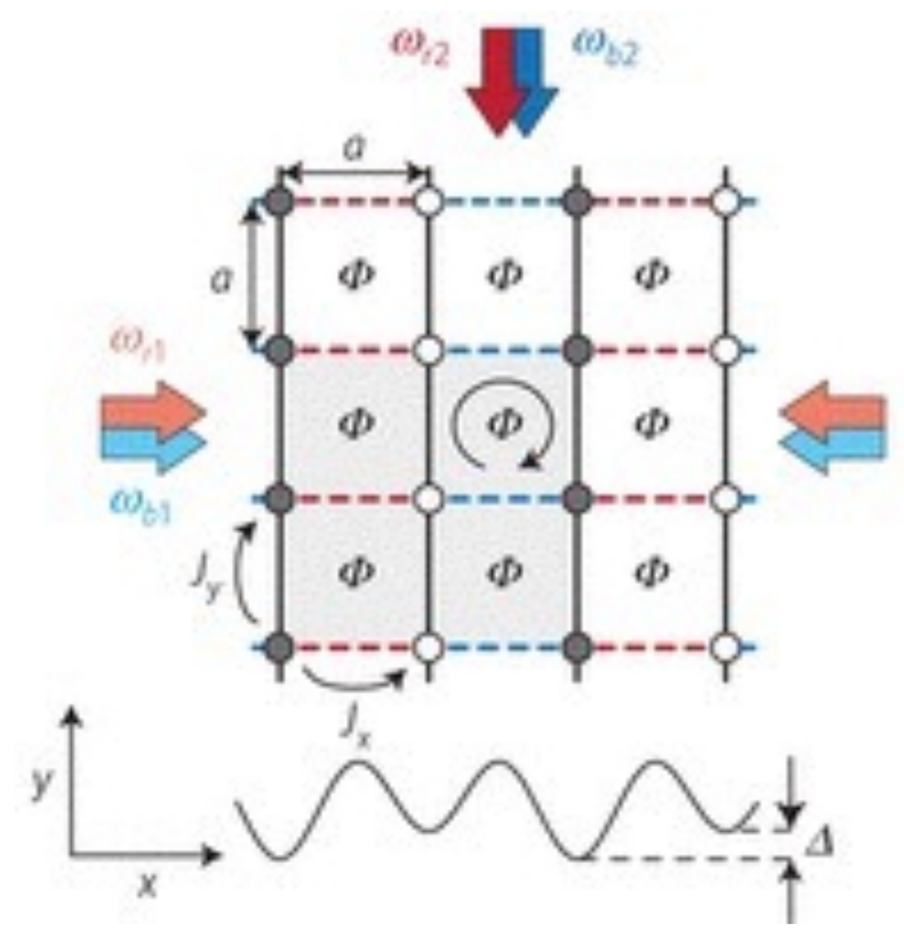


Simplest non-equilibrium setting: what can happen?

Engineer new states out of equilibrium?



Wang et al (Gedik group)
Science (2013)



Aidelsburger et al (Bloch group)
Nature (2014)

Outline

Steady states of Floquet systems

Thermal

(infinite temperature)

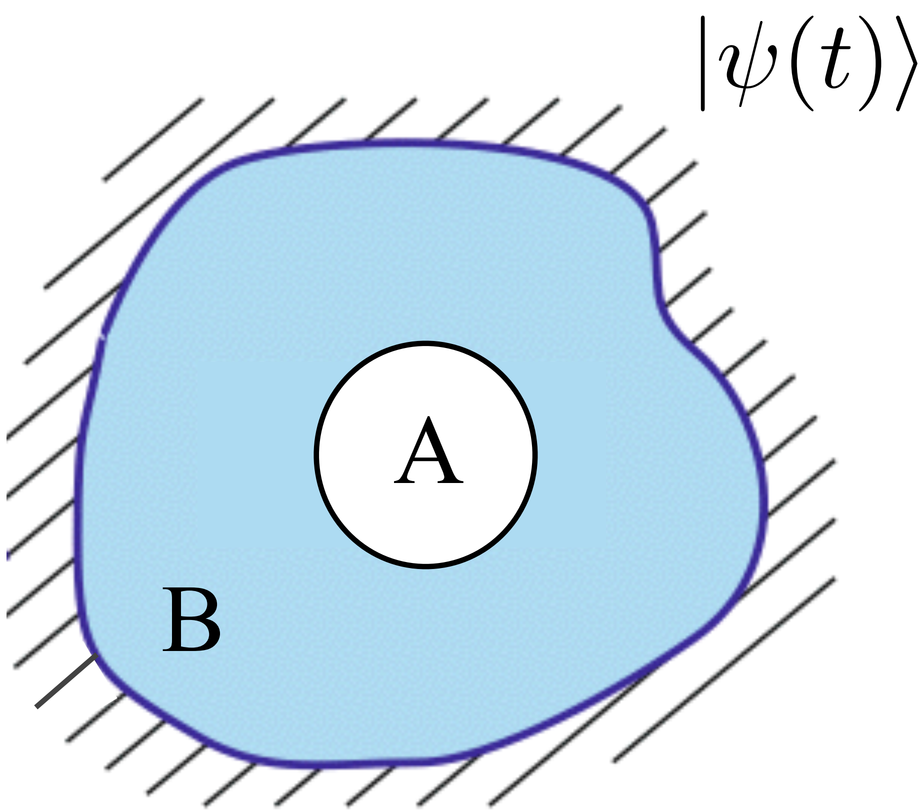
Time crystal

(period doubling in an “integrable” theory)

Many-body localized

(persistent local memory)

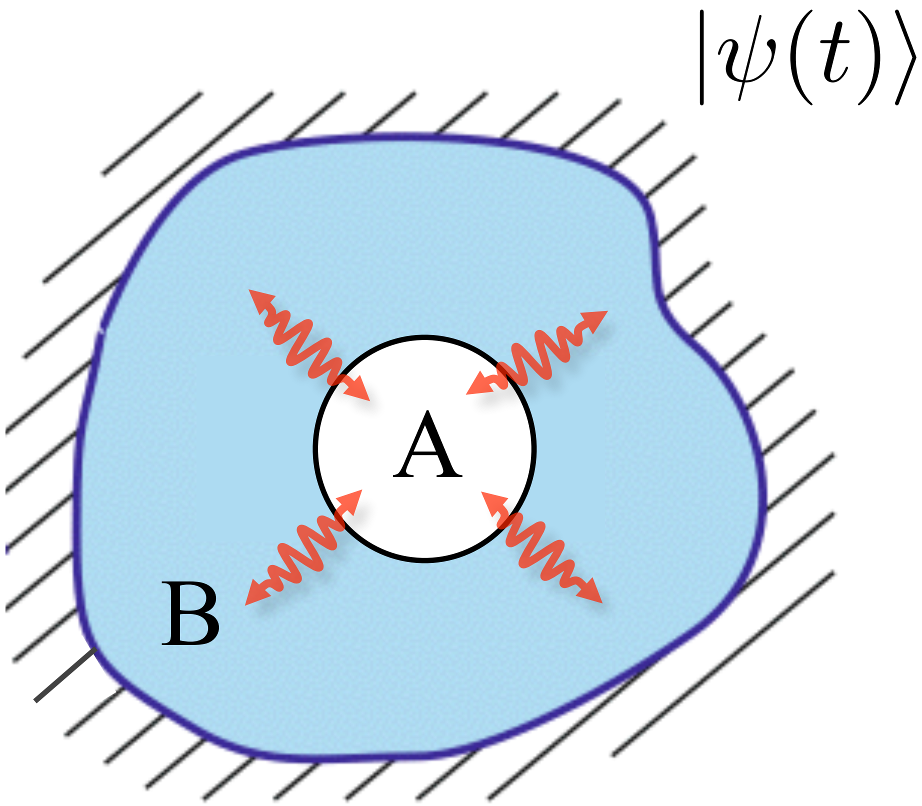
Thermalization in isolated systems



Local state

$$\rho_A(t) = \mathrm{Tr}_B |\psi(t)\rangle\langle\psi(t)|$$

Thermalization in isolated systems



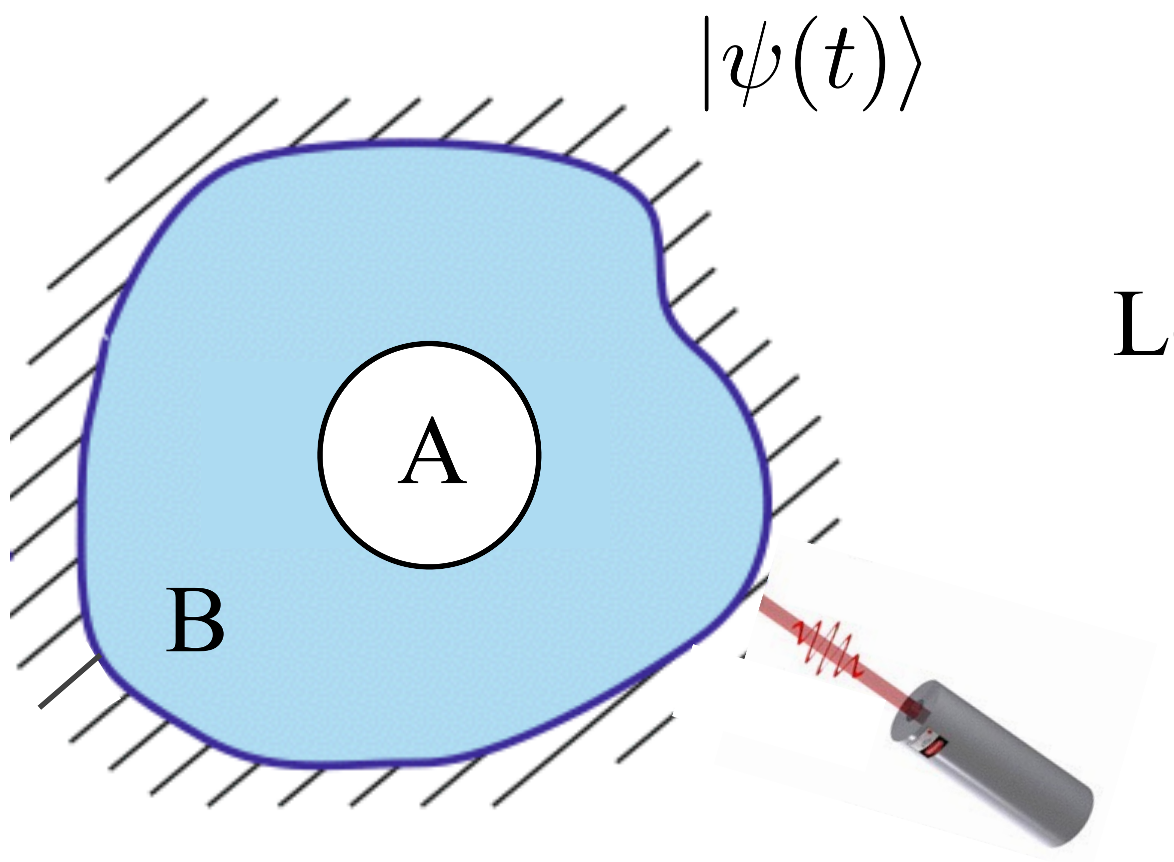
Local state

$$\rho_A(t) = \mathrm{Tr}_B |\psi(t)\rangle\langle\psi(t)|$$

No driving

$$\lim_{t\to\infty} \rho_A(t) = \frac{1}{Z}\mathrm{Tr}_B e^{-\beta H}$$

Thermalization in isolated systems



Local state

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No driving

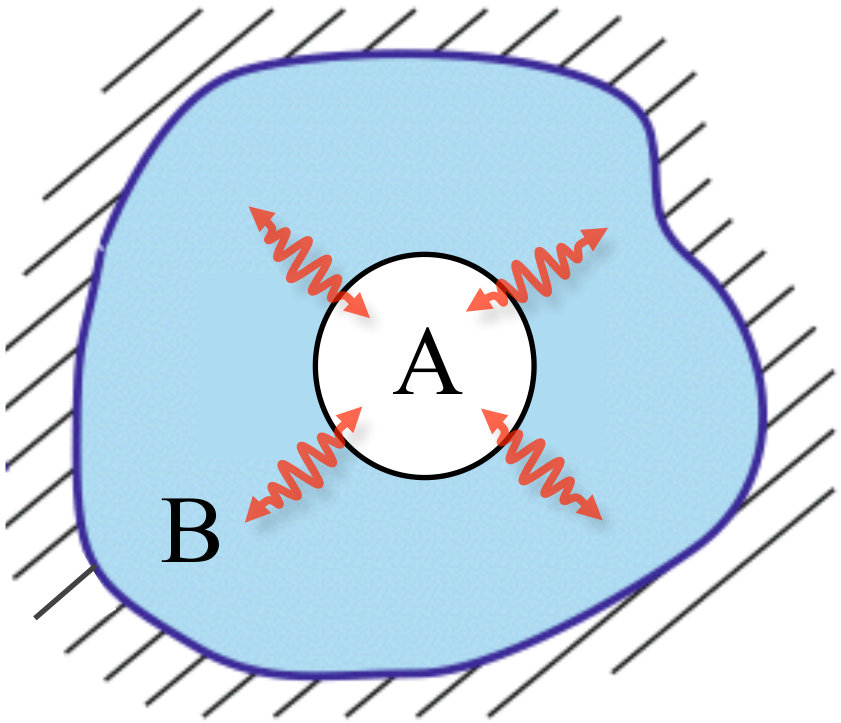
$$\lim_{t\to\infty} \rho_A(t) = \frac{1}{Z}\mathrm{Tr}_B e^{-\beta H}$$

With driving

$$\lim_{t\to\infty} \rho_A(t) = \frac{1}{Z}\mathrm{Tr}_B e^{-\not{\beta} H}$$

$\propto \mathbf{1}$

Eigenstate thermalization hypothesis (ETH)



For all eigenstates E_i
at inverse temperature β

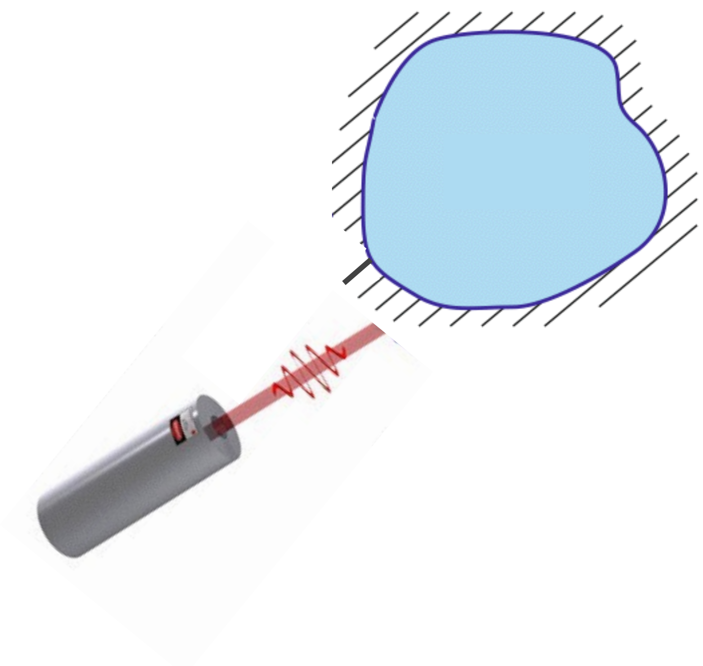
$$\rho_A = \mathrm{Tr}_B |E_i\rangle\langle E_i| = \frac{1}{Z}\mathrm{Tr}_B e^{-\beta H}$$

$$H|E_i\rangle = E_i|E_i\rangle$$

ETH $\Rightarrow$ thermalization
Generically thermalization seems to $\Rightarrow$ ETH

Deutsch (1991)
Srednicki (1994)

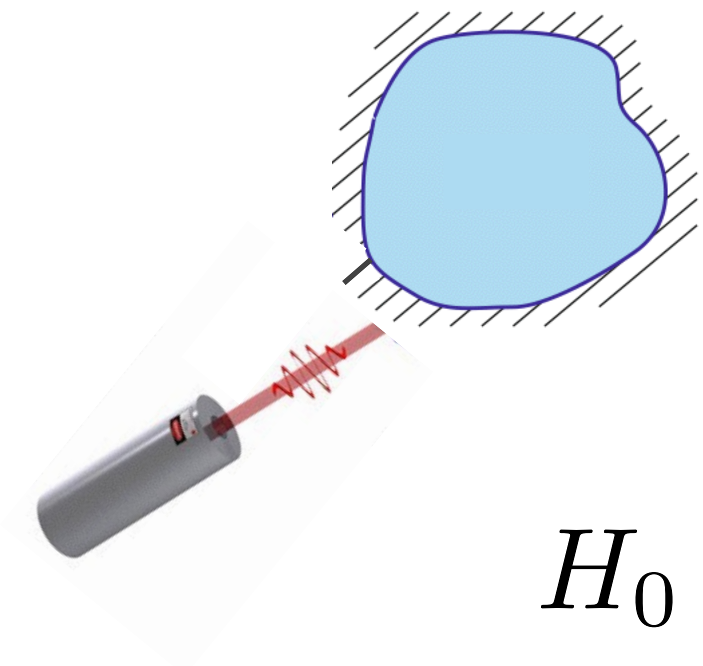
Driven eigenstates



$$H(t) = H_0 + V\cos(\omega t) H_1$$

Floquet/periodic evolution: $$U(T) = Te^{-i\int_0^T H(t)dt'}$$

Driven eigenstates



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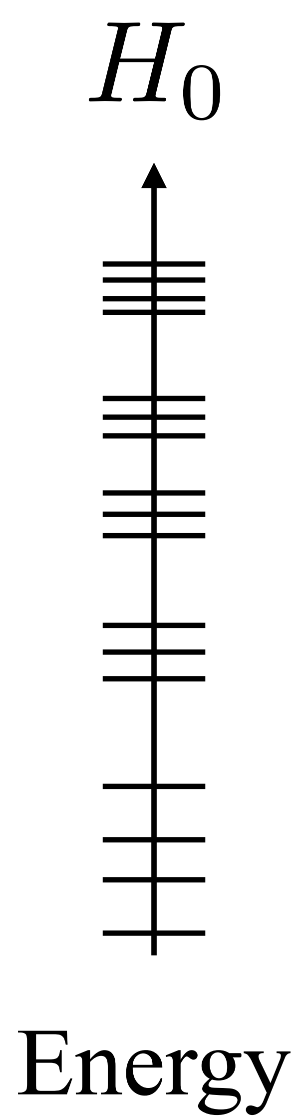
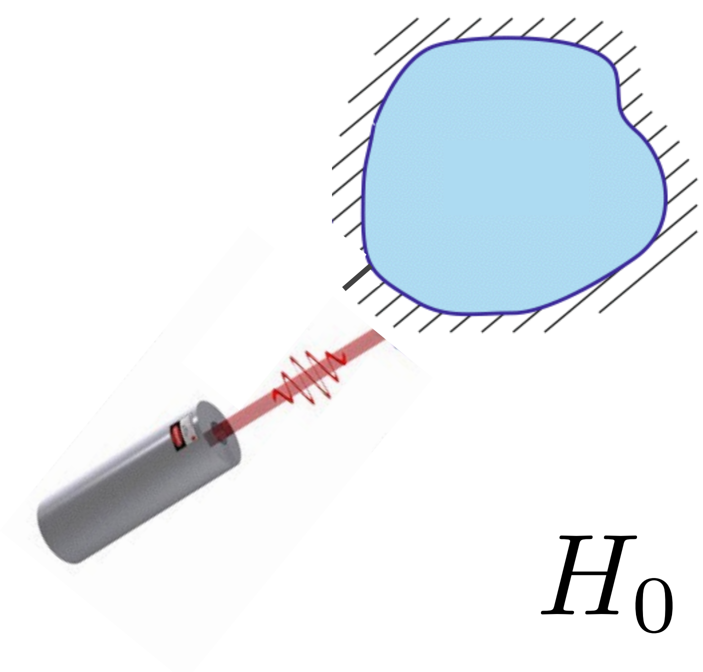


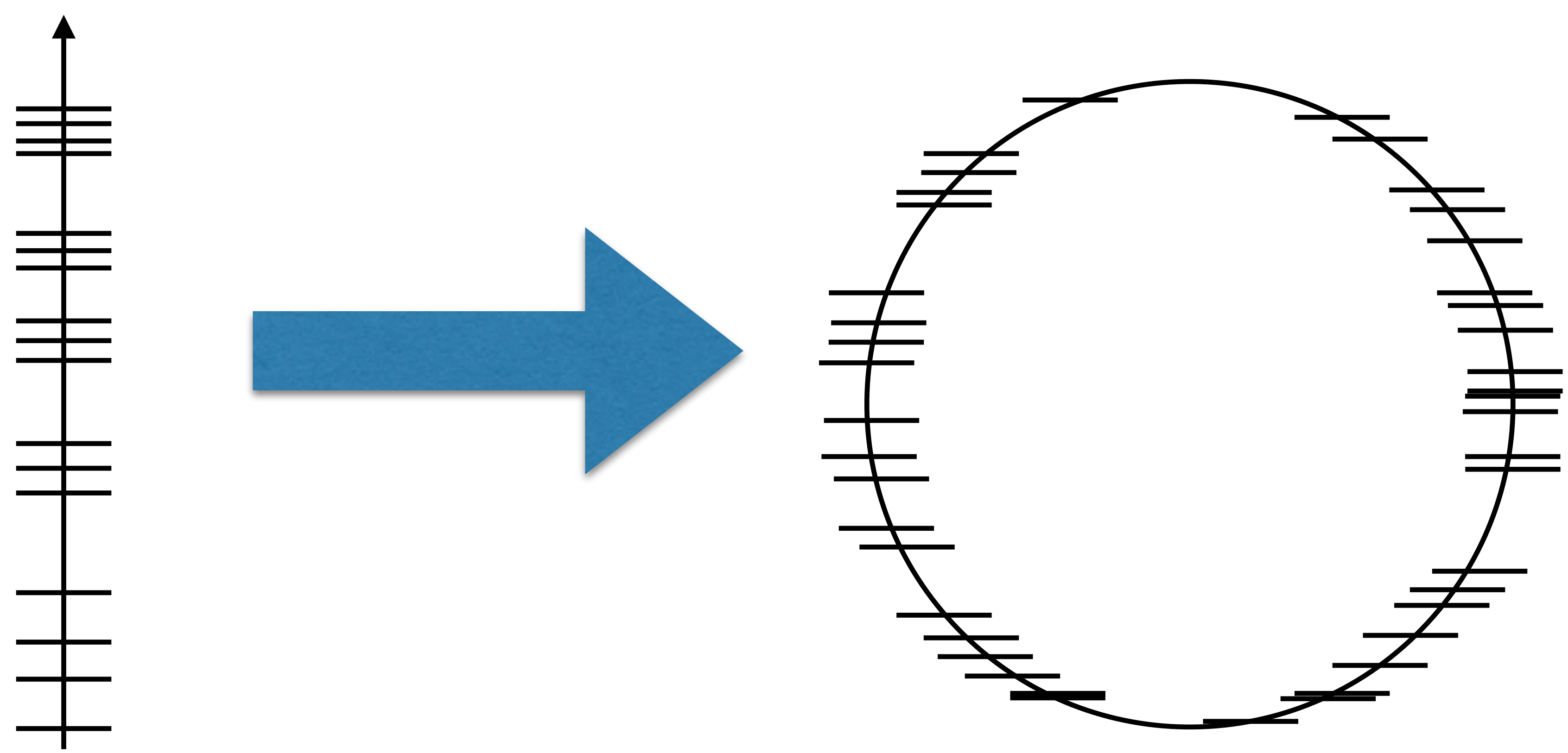
Driven eigenstates



$$H(t) = H_0 + V\cos(\omega t)H_1$$

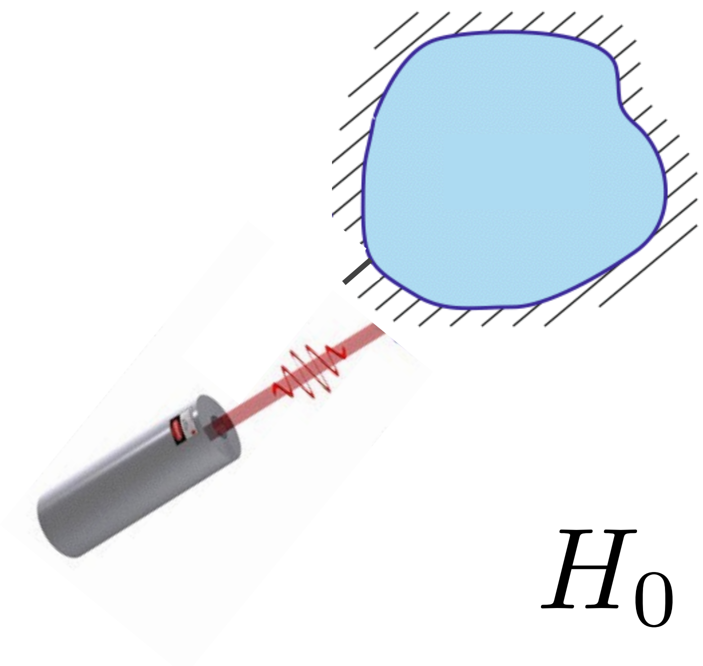
Floquet/periodic evolution: $U(T) = Te^{-i\int_0^T H(t)dt'}$

H_0 $\quad$ $U(T)$



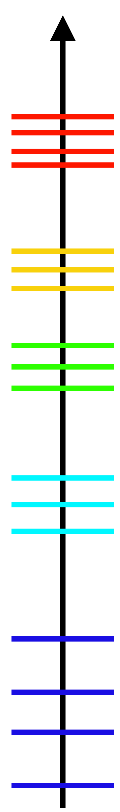
Energy $\quad$ Floquet Quasi-energy

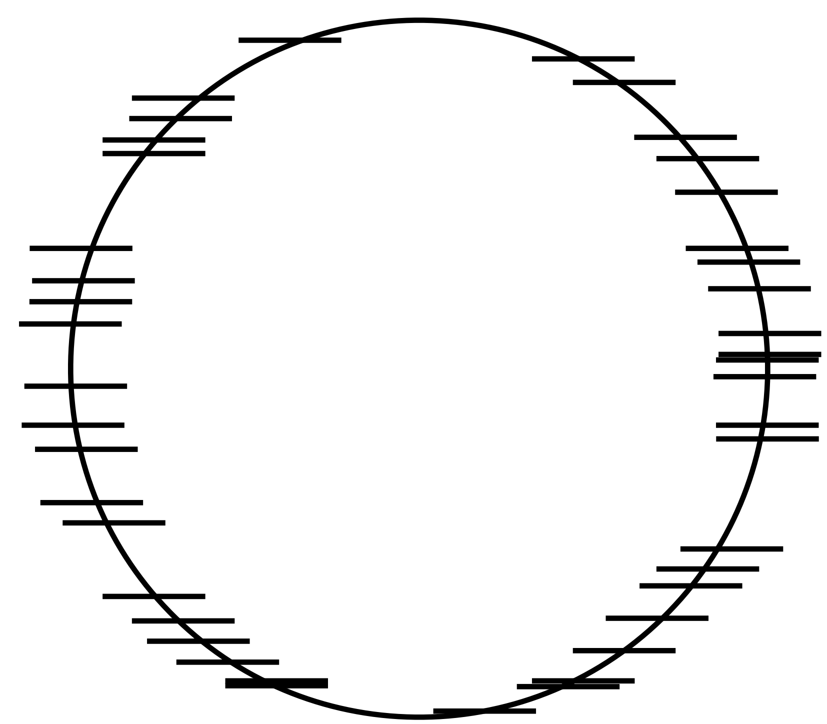
Driven eigenstates



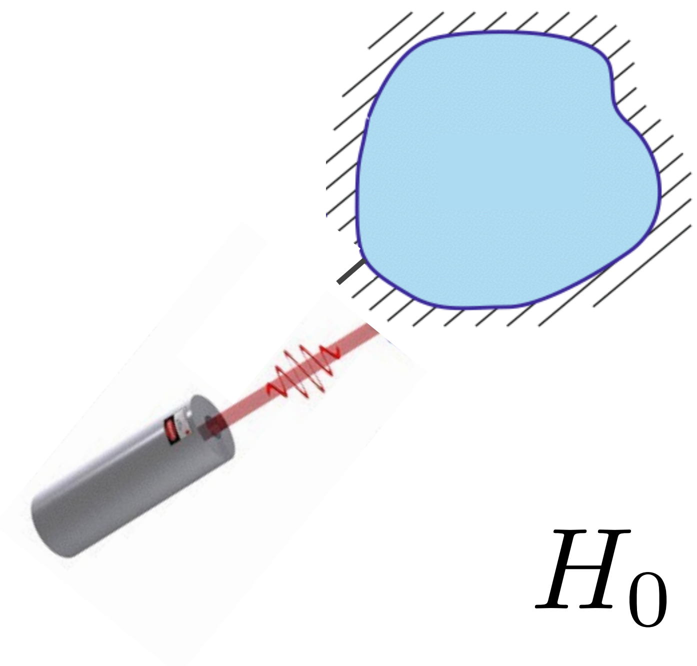
$$H(t) = H_0 + V\cos(\omega t) H_1$$

Undriven eigenstates ETH





Driven eigenstates

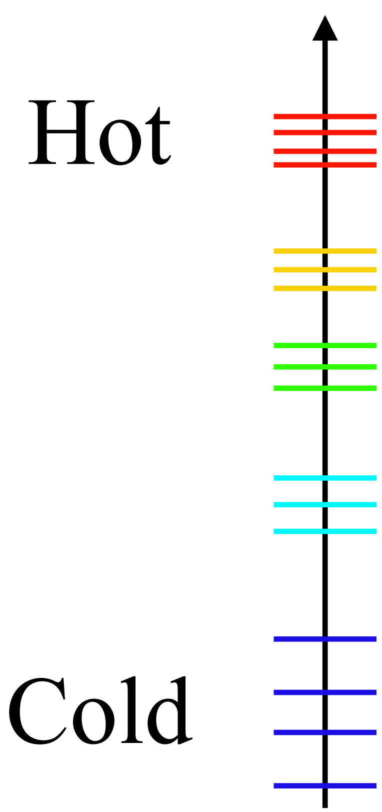


$$H(t) = H_0 + V\cos(\omega t)H_1$$

Undriven eigenstates ETH

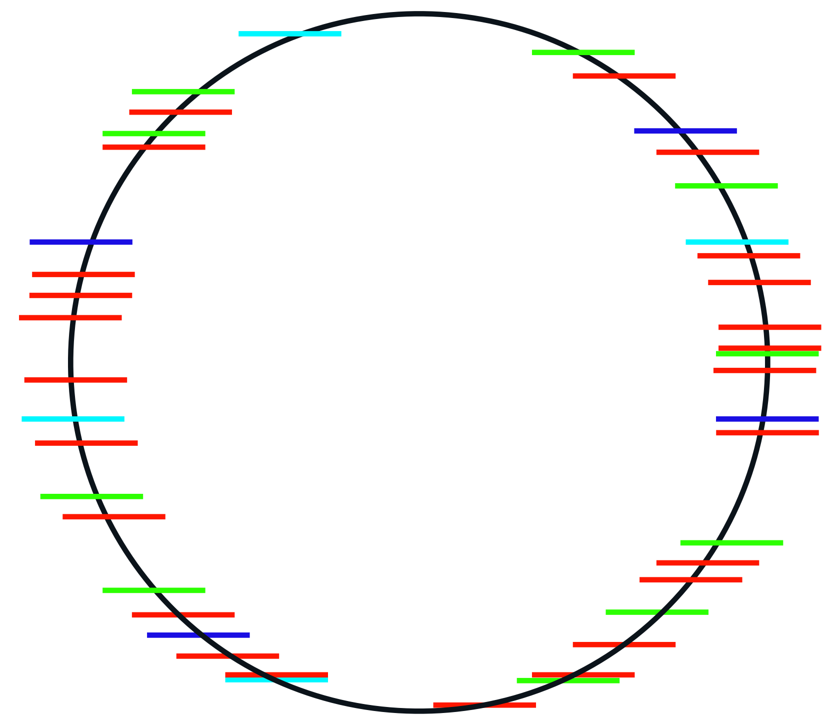
Driven eigenstates ?

H_0



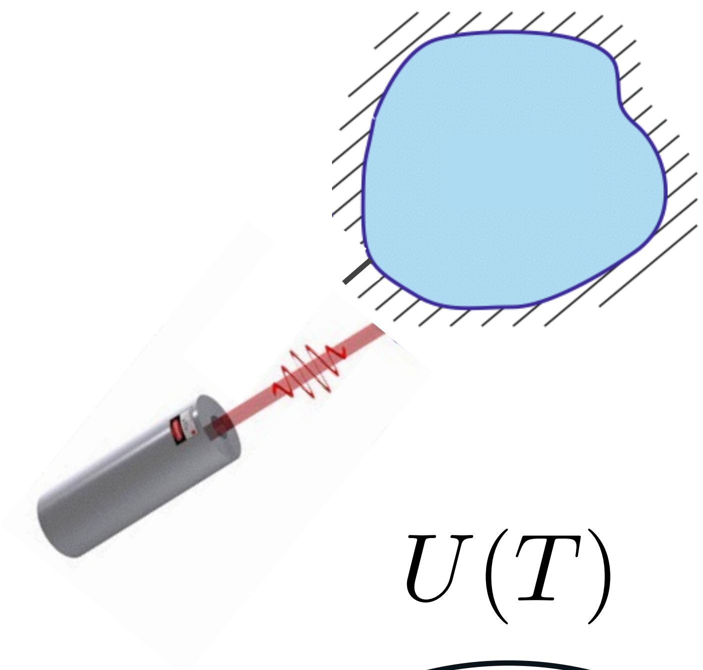
Energy

$U(T)$



Floquet Quasi-energy

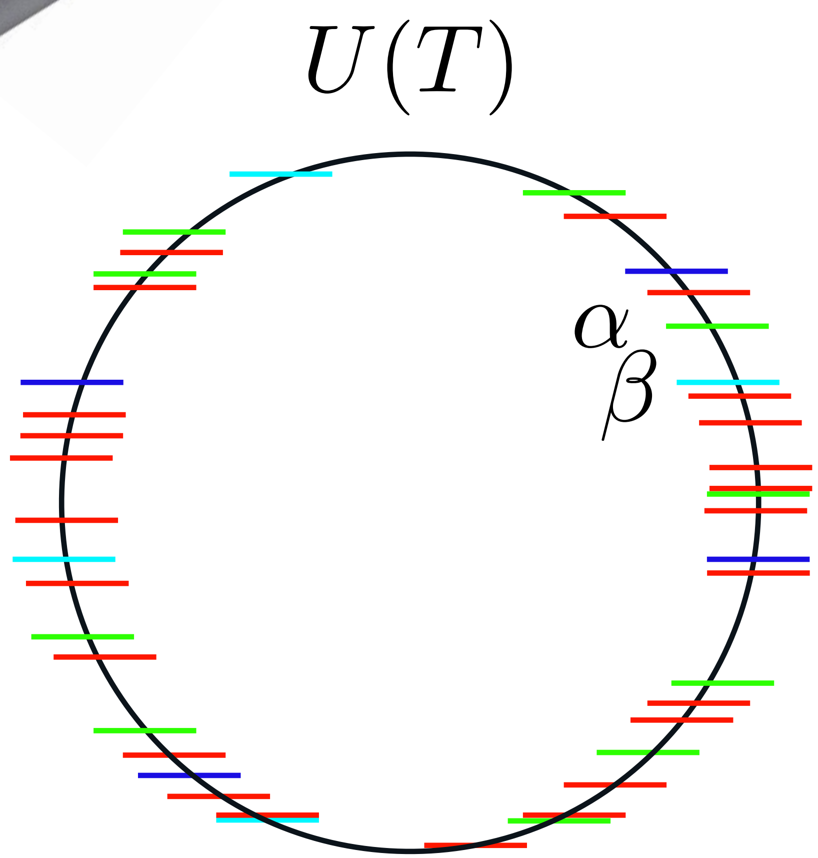
Driven eigenstates



$$H(t) = H_0 + V\cos(\omega t) H_1$$

Undriven eigenstates ETH

Driven eigenstates ?



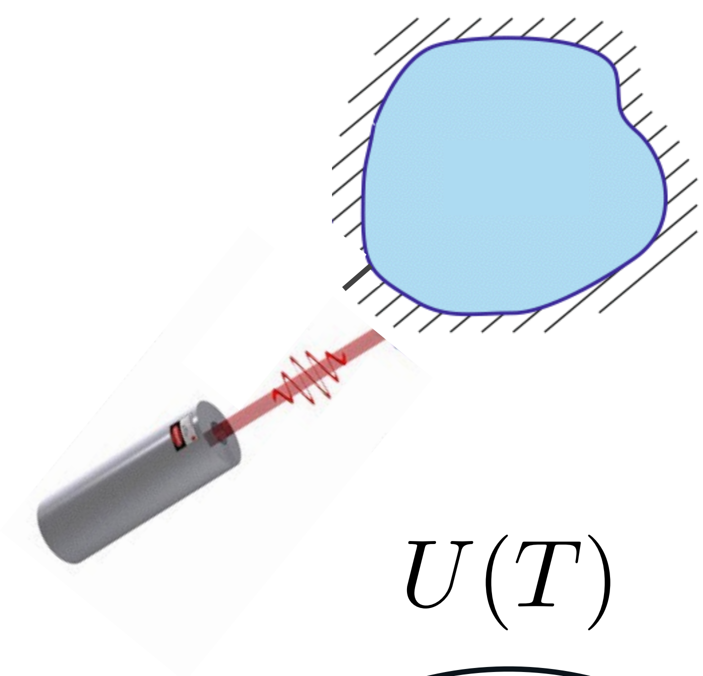
Floquet Quasi-energy

Local drive: $\langle E_\beta | U(T) | E_\alpha \rangle \sim \dfrac{1}{\sqrt{2^L}}$

Deutsch (1991)
Srednicki (1994)

Ponte, **AC**, Papic, Abanin (2015)

Driven eigenstates



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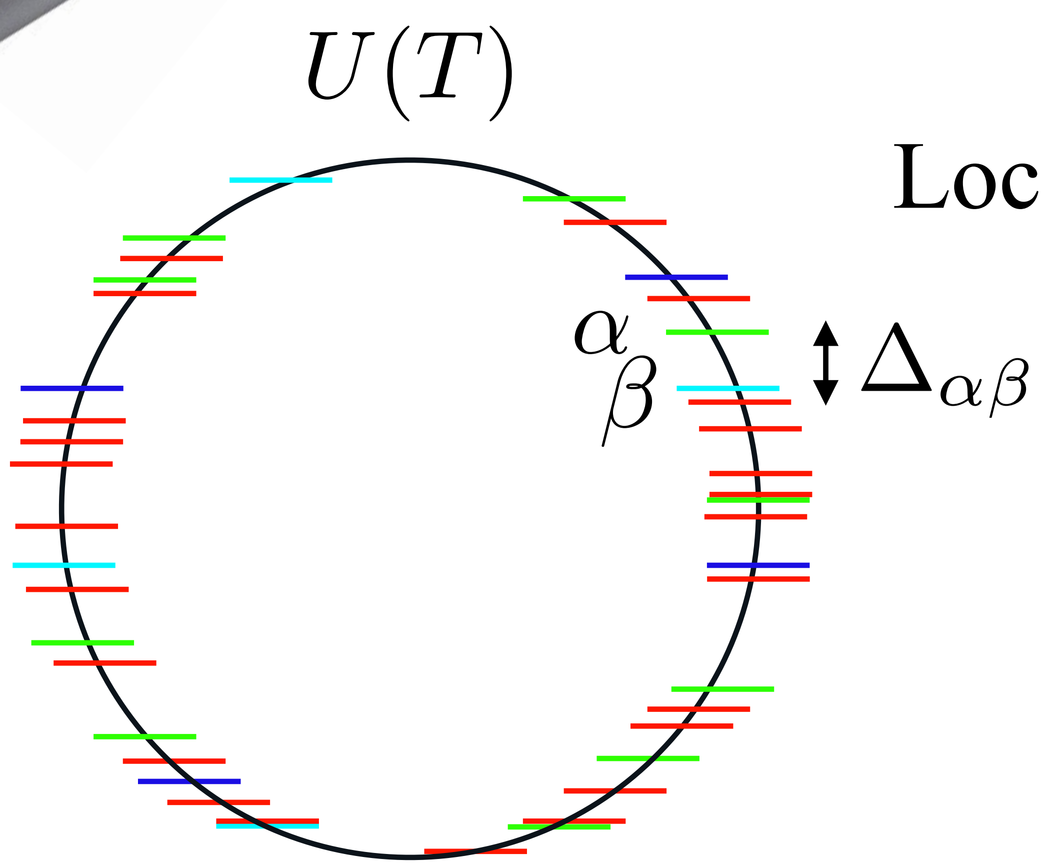
Undriven eigenstates ETH

Driven eigenstates ?



Floquet Quasi-energy

Local drive: $\langle E_\beta | U(T) | E_\alpha \rangle \sim \frac{1}{\sqrt{2^L}}$

$$\Delta_{\alpha\beta} \sim \frac{1}{2^L}$$

Deutsch (1991)
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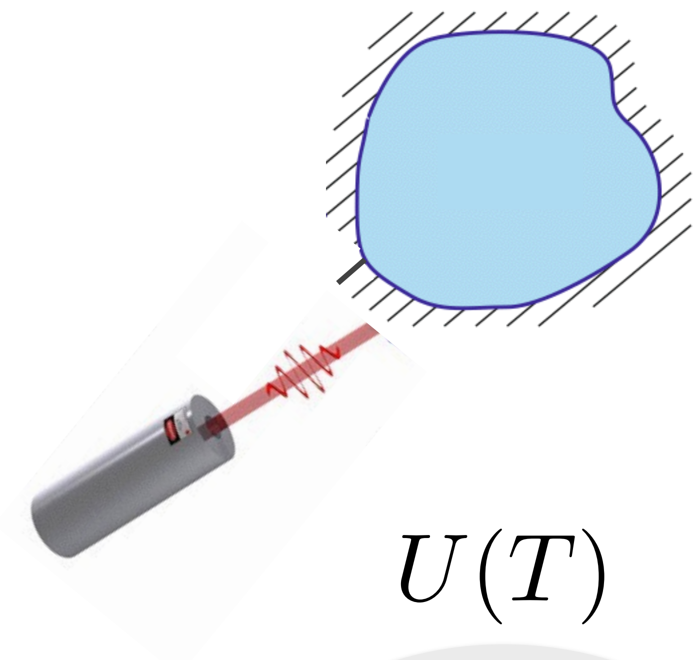
Driven eigenstates



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Undriven eigenstates ETH

Driven eigenstates ?

$U(T)$

Floquet Quasi-energy

Local drive: $\langle E_\beta | U(T) | E_\alpha \rangle \sim \frac{1}{\sqrt{2^L}}$

$$\Delta_{\alpha\beta} \sim \frac{1}{2^L}$$

Floquet eigenstates mix all temperatures!

Deutsch (1991)
Srednicki (1994)

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(period doubling in an "integrable" theory)

Many-body localized

(persistent local memory)

Interacting driven bosons

Driven O(N) model

$$H(t) = \frac{1}{2}\int d^d x(|\Pi|^2 + |\nabla\Phi|^2 + r(t)|\Phi|^2 + \lambda|\Phi|^4)$$

$$r(t) = r_0 - r_1 \cos(\gamma t)$$

AC, Sondhi (2015)

Interacting driven bosons

Driven O(N) model

$$H(t) = \frac{1}{2} \int d^d x (|\Pi|^2 + |\nabla \Phi|^2 + r(t)|\Phi|^2 + \lambda |\Phi|^4)$$

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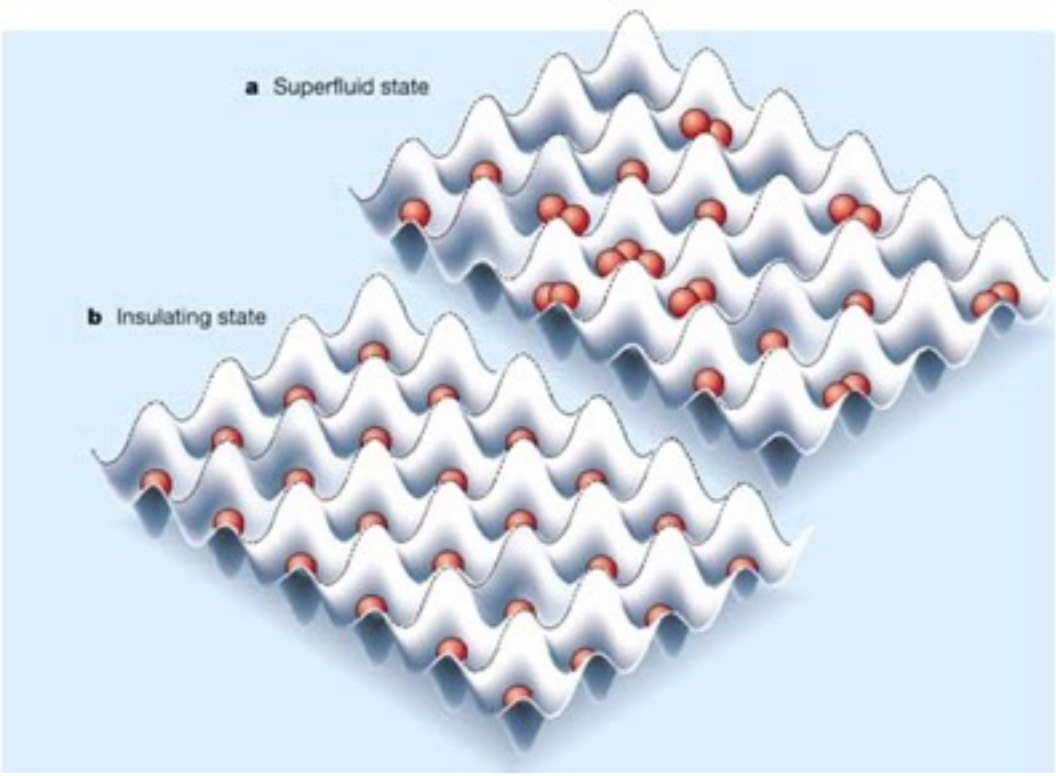
O(2) version:

Near transition from Mott insulator to superfluid

r(t): modulating tunneling



AC, Sondhi (2015)

Interacting driven bosons

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$$H(t) = \frac{1}{2}\int d^dx(|\Pi|^2 + |\nabla\Phi|^2 + r(t)|\Phi|^2 + \lambda|\Phi|^4)$$

$$r(t) = r_0 - r_1\cos(\gamma t)$$

Equilibrium: canonical model for symmetry-breaking

Analytical control in the large-N limit

Self-consistent classical equations

$$\left(\frac{d^2}{dt^2} + |\vec{k}|^2 + r(t) + N\lambda\int^{\Lambda}\frac{d^dk}{(2\pi)^d}|f_{\vec{k}}(t)|^2\right)f_{\vec{k}}(t) = 0$$

AC, Sondhi (2015)

Interacting driven bosons

$\vec{k}_1$ $\vec{k}_2$ $\vec{k}_3$

$$\omega_{\vec{k}}(t)^2 = |k|^2 + r(t) + r_{eff}(t)$$

Feedback term

Feedback term prevents parametric resonance

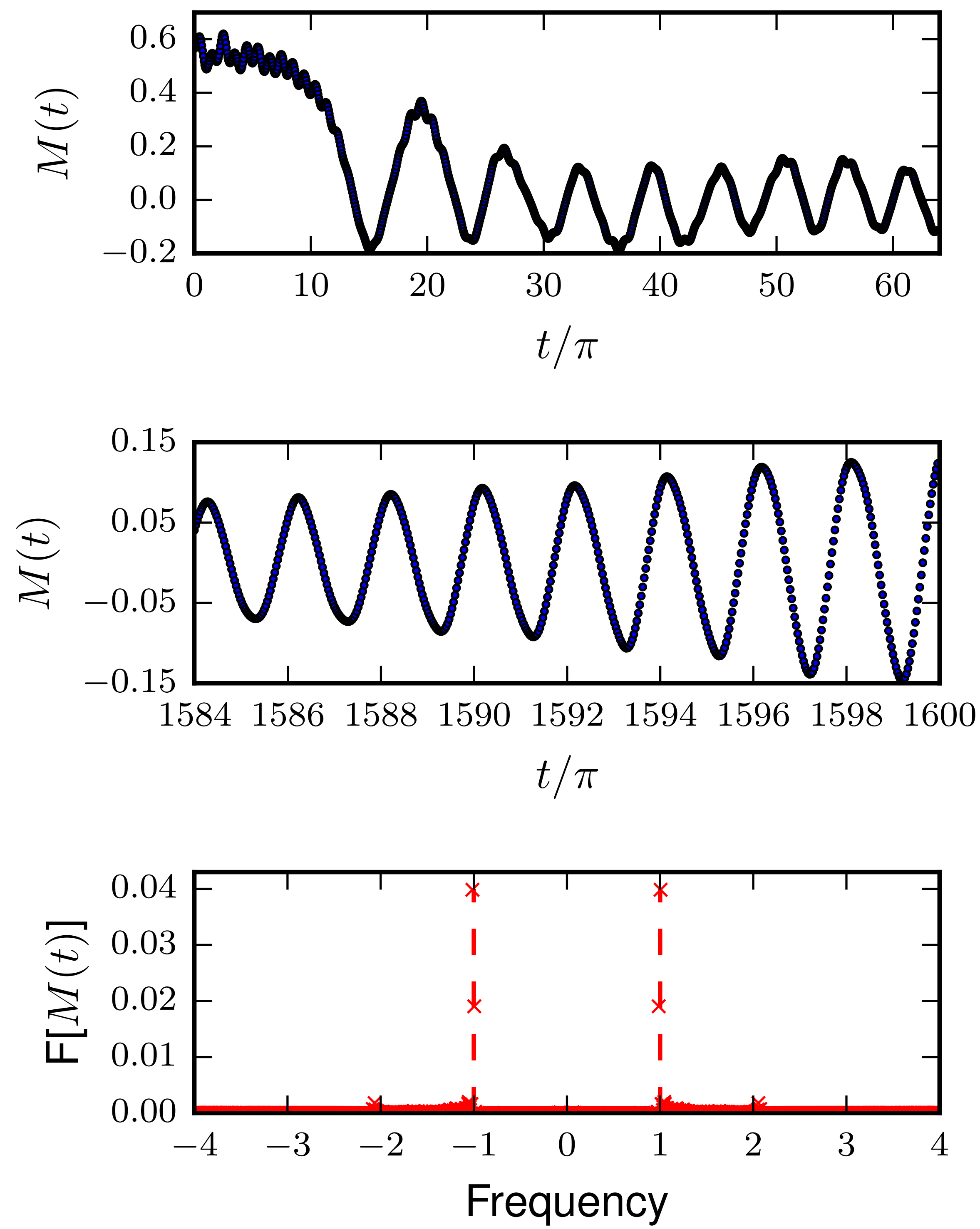
Steady state: finite correlations with structure

“Integrable”: unknown generalized Gibbs ensemble

AC, Sondhi (2015)

Period doubling in the driven ferromagnet

Drive period $= \pi$



AC, Sondhi (2015)

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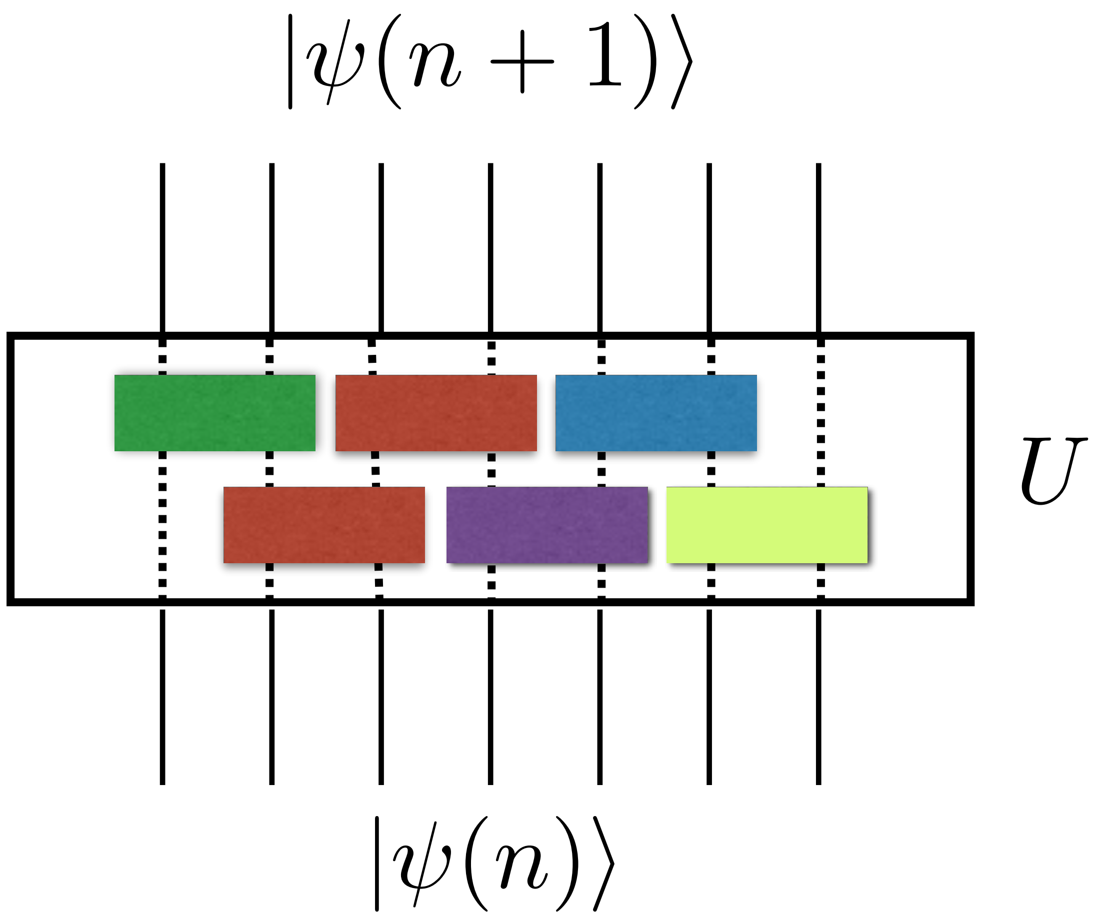
Periodic circuits



$$|\psi(n)\rangle = U^n|\psi(0)\rangle$$

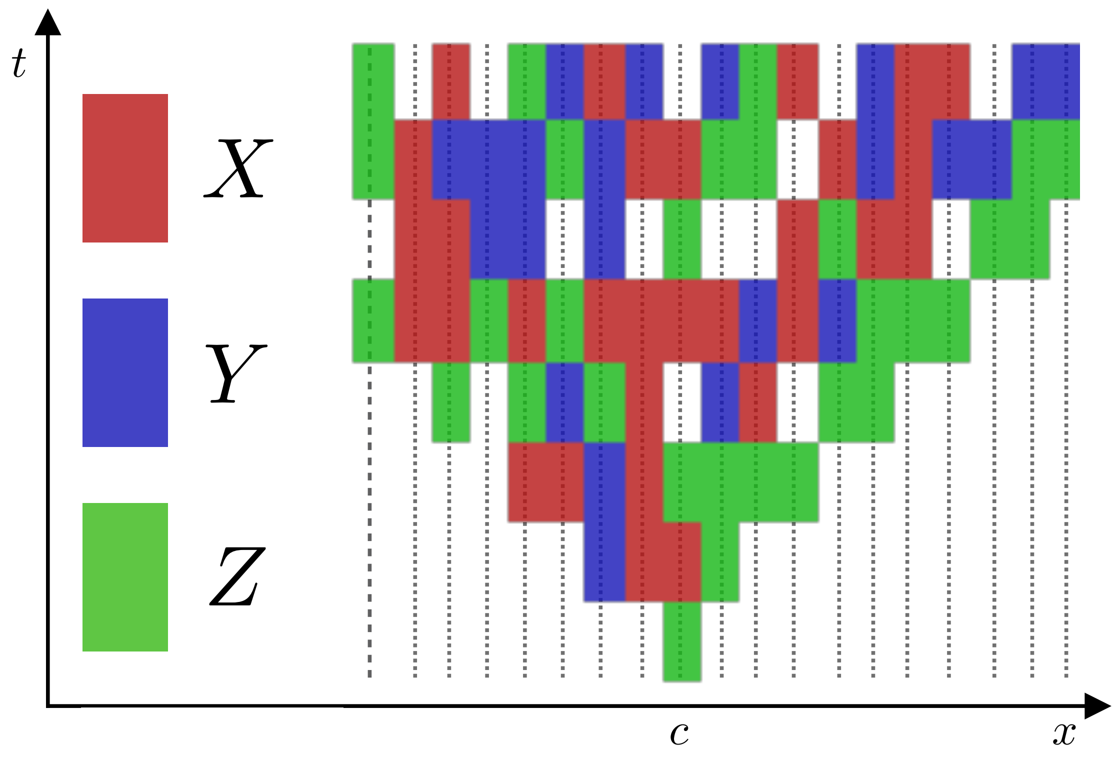
Floquet evolution without H(t)!

Clifford circuits

- Clifford gates: Hadamard, Phase, CNOT
- Efficiently simulable (poly(N) time for N qubits)
  - $U^\dagger (X_1 \otimes Z_2 \otimes \ldots 1_N) U = Y_1 \otimes X_2 \otimes \ldots Z_N$
- Can entangle
- Infinite temperature locally?

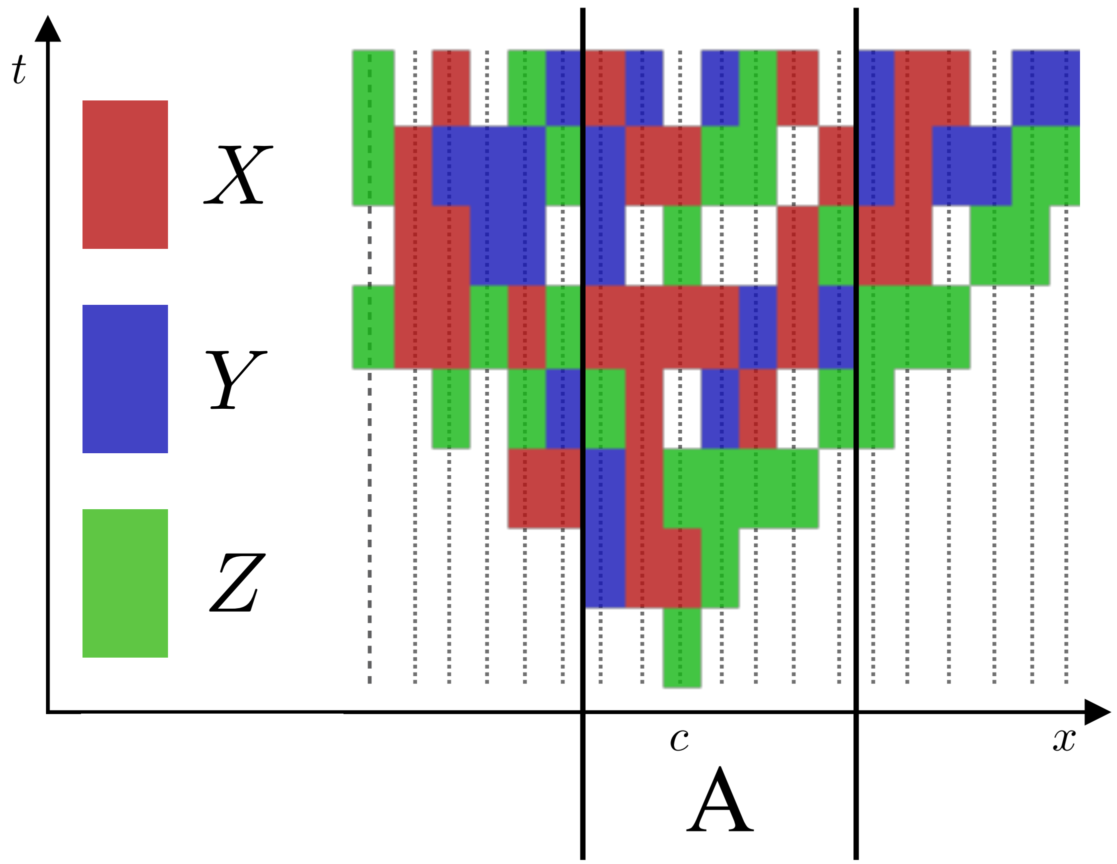
Gottesman-Knill (1996)
Aaronson & Gotteman (2004)

Thermalization

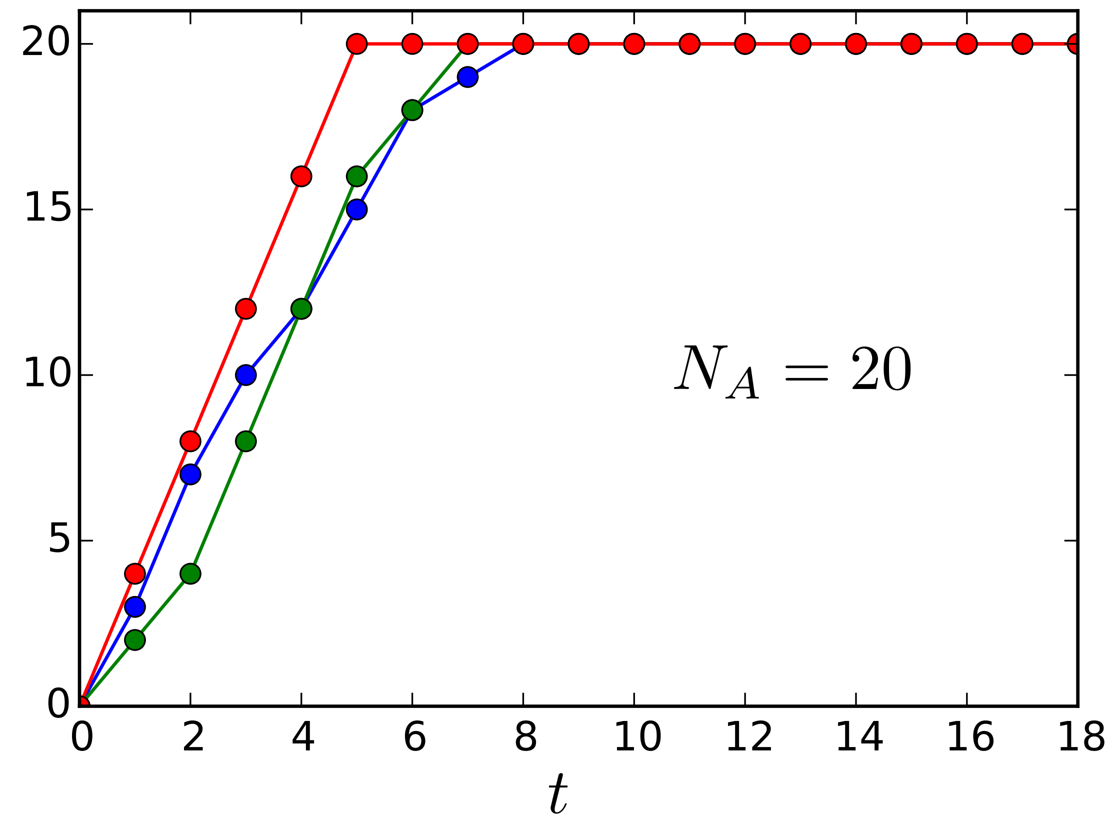


AC, C. R. Laumann (2015)

Thermalization



$$S_A(t) = -\mathrm{Tr}\rho_A \log_2 \rho_A$$



AC, C. R. Laumann (2015)

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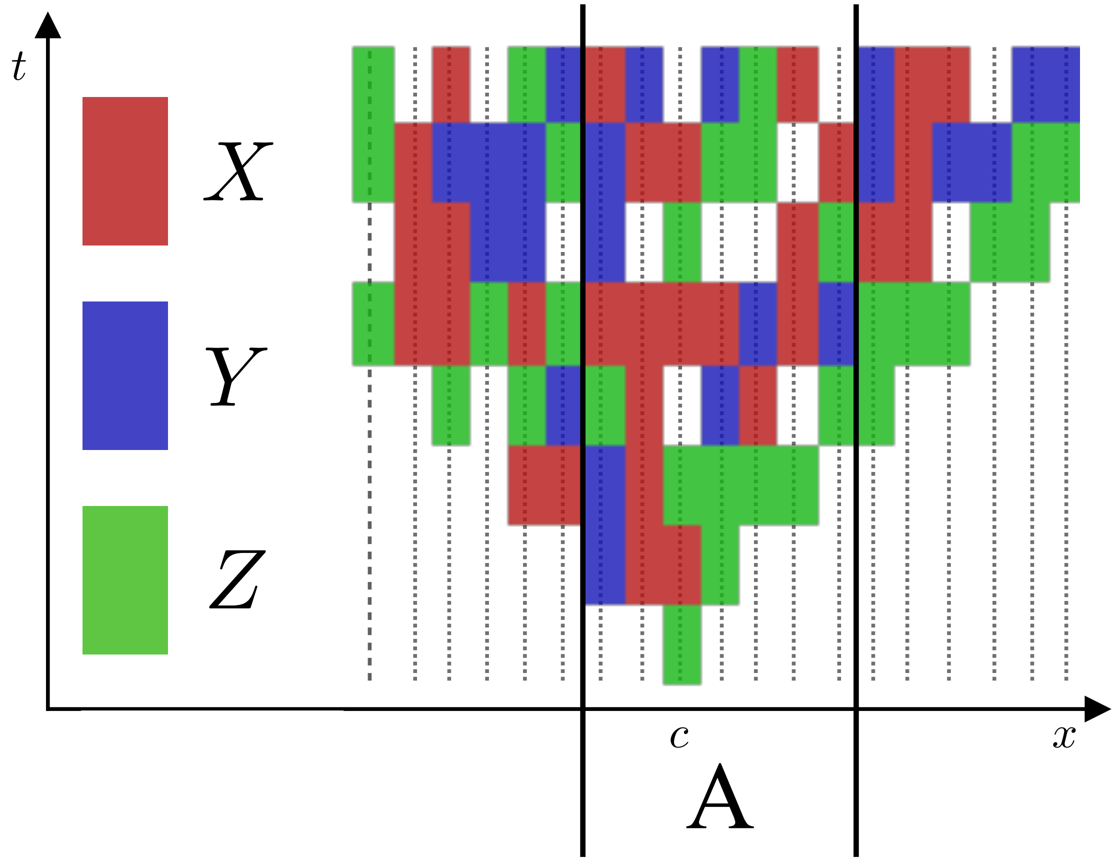
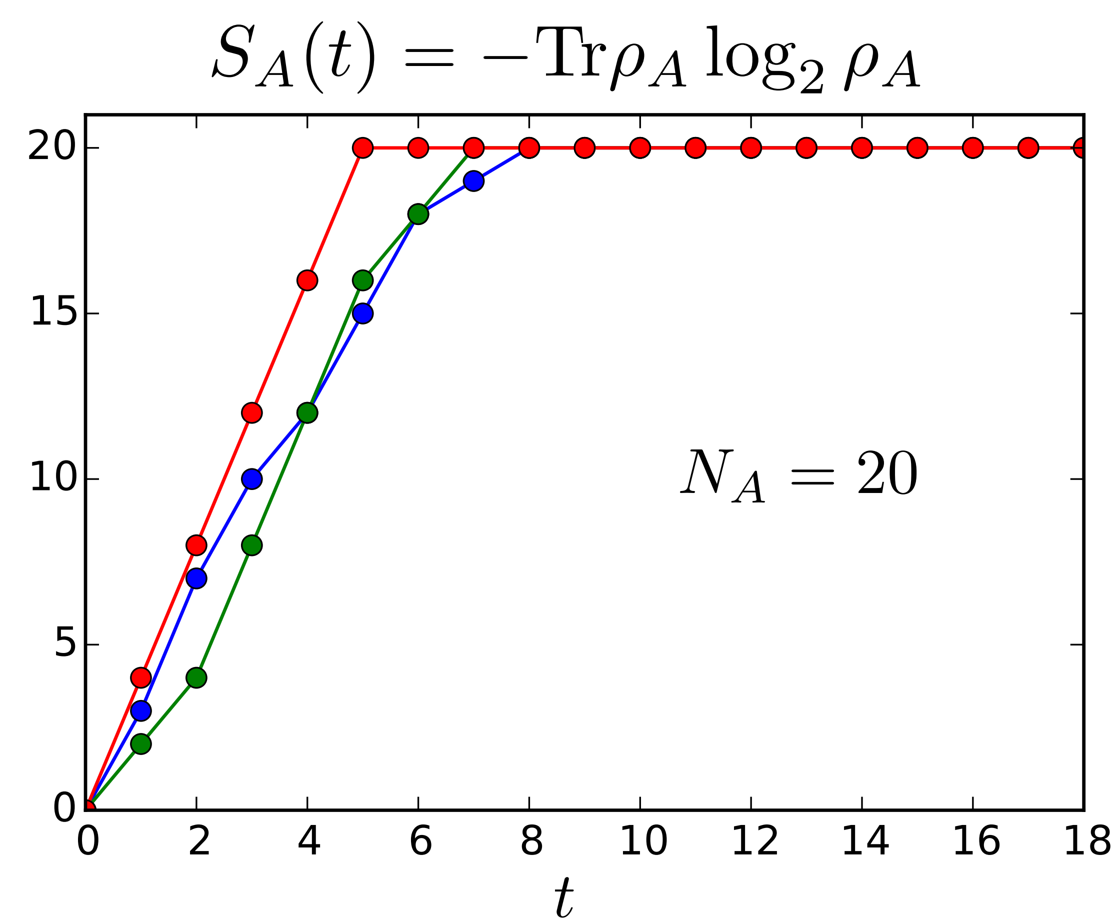
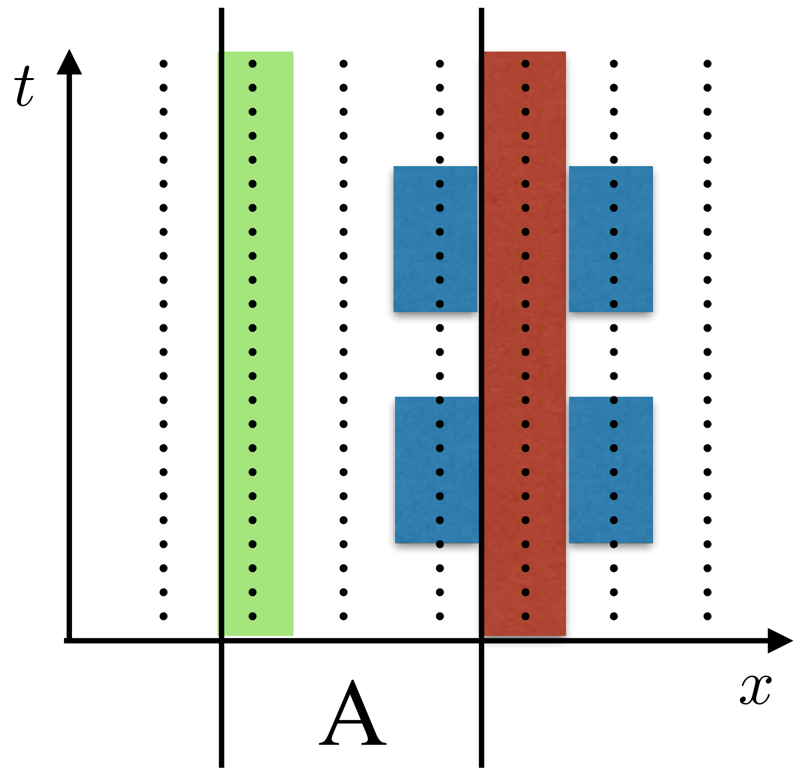


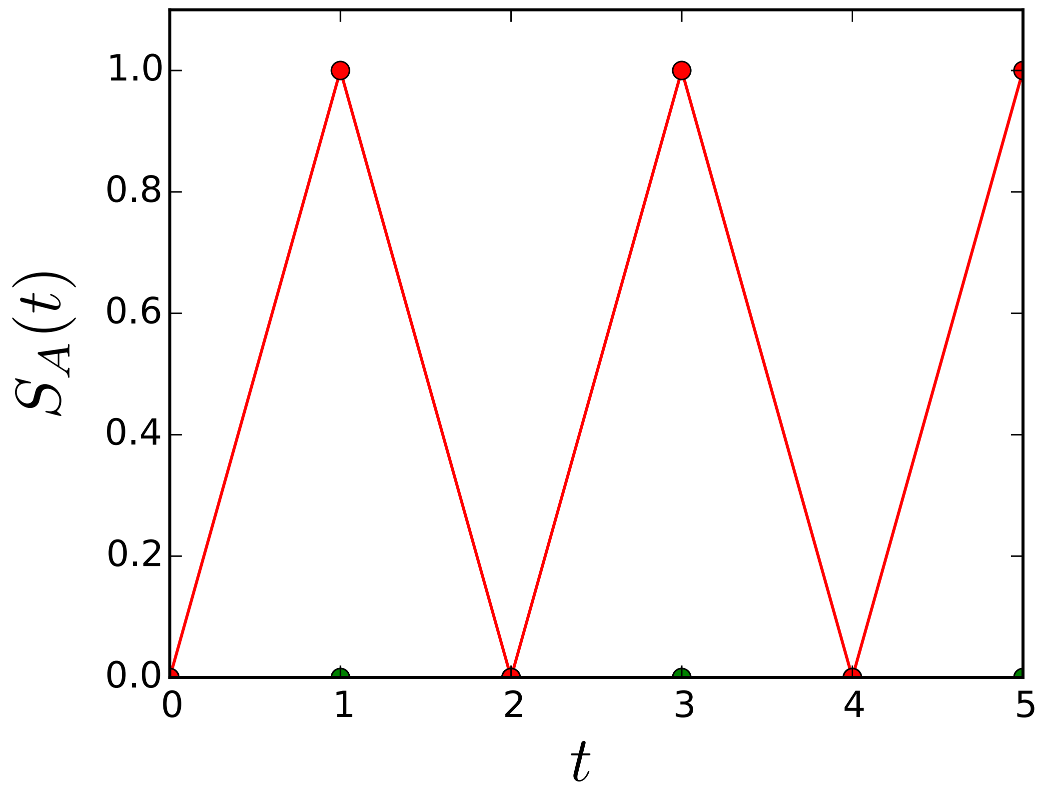


- Operator support grows in time
  - $\rho_A = 1$ for $t > vN_A$
- Simulable system that thermalizes!

AC, C. R. Laumann (2015)

Localization





- Strictly local integrals of motion: Z_i
- Block spread of information
- Transition to thermalization: percolation of operator support

AC, C. R. Laumann (2015)

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Outline

Steady states of Floquet systems

Thermal

(infinite temperature)

?

Pre-thermal classification?

Time crystal

(period doubling in an “integrable” theory)

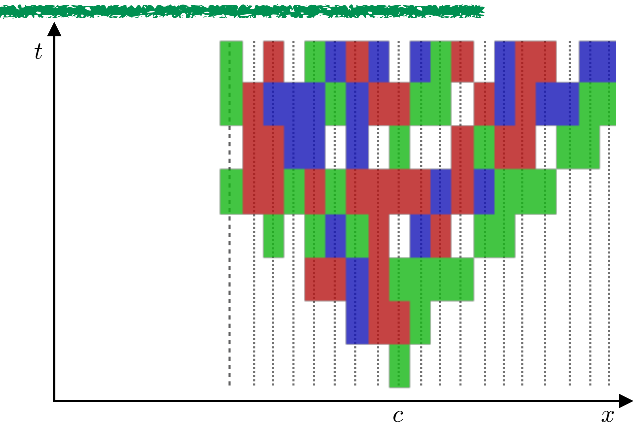
?

With dissipation?

Many-body localized

(persistent local memory)

Thank you



Thank you to my collaborators

Dima Abanin, Chris Laumann,
Zlatko Papic, Pedro Ponte & Shivaji Sondhi

&

Thank you for your attention!

BOSTON UNIVERSITY